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THE MATHEMATICS TEACHER

Volume XXXVII Number 2

Edited by William David Reeve

Mathematics that Functions in War and Peace*

By H. C. CHRISTOFFERSON
Miami University, Oxford, Ohio

THE Army and Navy in their training courses as shown in recent issues of THE MATHEMATICS TEACHER, have emphasized greatly the practical and educational values of mathematics. In the V-12 and A-12 programs planned for prospective officers, mathematics and science make up over half of the work. In the training of inductees persisent stress is placed upon knowledge of basic mathematical skills and abilities. It is regrettable that it took a war to make schools realize the large values inherent in mathematical education. Our complex civilization is becoming constantly more and more scientific and mathematical, yet both mathematics and science had been receiving before the war a relatively smaller and smaller place in the curriculum.

As mathematics teachers we were prone to blame our educational leadership for this emphasis. Many of these leaders were school administrators who were former mathematics teachers. Some held tenaciously to requirements in arithmetic, algebra and geometry, others caught the vision of the value in the social studies, and lost their sense of value in the exact sciences. Yet much of the blame for the near eclipse of mathematics must be placed at our own doors as mathematics teachers. We were too content with a formal discipline developed for the gifted few, and refused or failed to temper and adjust it for the many. Our textbook writers often seemed more concerned with maintaining the status quo, with protecting their vested interests, with defending the old values, than with making adjustments to new situations, with selecting and promoting new and needed techniques, with recognizing the need for a meaningful and functional mathematics serving modern society.

Analysis of past failures can have value only if it throws light on future problems and assists in their solution. When the war is over mathematics will soon slip back into a secondary position in our school curriculum unless it can demonstrate its place and function in the lives of people who learn it. It is the purpose of this discussion to stress some aspects of teaching and of subject matter which should help to keep mathematics meaningful and functional and therefore tend to insure that it serve youth effectively.

The first of these aspects of mathematics teaching to which I wish to direct your attention is teacher enthusiasm, the spirit of magic and adventure that is inherent in algebra particularly, the vision of the power of an equation, of the time

* An address given at the Mathematics Section of North Western Ohio Education Association in Toledo, Ohio, October 29, 1943.

and energy saving machinery in a formula, of the wide applications of symbolic thinking to all situations in which one quantity is determined by one or more others. Teachers of mathematics so easily become formal, seem so easily to lose their sense of adventure and their ability to be enthused over the remarkable powers of arithmetic, algebra and geometry.

The second is a subject-matter consideration. The equation and formula have long been the heart of algebra. Yet their interrelationship and their dependence upon the more formal portions of algebra have not always been clear nor adequately emphasized. It shall be my function here to stress this interrelationship, to show the use of a formula as an ultimate solution for a problem and in the doing of this to point out the possibilities for adventure, for enthusiasm, for appreciation of the "super" powers and almost magical properties of the equation and formula in solving problems. I shall begin with some very simple problems and proceed to more complex ones as illustrations of these two ideas.

ILLUSTRATION 1

An army aviator is sent out on a scouting flight. He has a four-hour supply of gasoline, and has computed that his ground speed out with the wind is 150 mph and back it is 100 mph. How long can he fly out and yet return in safety?

Very erroneous thinking would deduce that if he has a four-hour supply of gas, he can fly out 2 hr. and back 2 hr. Yet a little checking reveals that in 2 hr. he would be 300 mi. out, and then in another 2 hr. he will fly back only 200 mi. Therefore, he will crash or be forced to land 100 mi. or 1 hr. away from the starting point.

A little deeper thinking but still in error, would conclude that if the two groundspeeds are 150 and 100 mph respectively the average speed would be 125 mph. Then in 4 hr. one would average 125 mph or one would travel 500 mi. Half of this would be 250 mi. out and 250 mi. back. This erroneous solution is better than the first, yet a little checking again reveals its crash and death effects. At 150 mph to travel 250 mi. would require $1\frac{2}{3}$ hr. or 1 hr. 40 min. To return at 100 mph would require $2\frac{1}{2}$ hr., which added to 1 hr. 40 min., makes 4 hr. 10 min. of total flying time. Again, if he had exactly 4 hours of gasoline, he would crash or be forced to land 10 min. short of his destination. At 100 mph in 10 min. he would have flown $16\frac{2}{3}$ mi., therefore, he would still be out or possibly dead, about 17 mi. from port.

The human mind through centuries has developed a remarkable machine for solving elusive quantitative problems. Suppose we feed this problem material into the problem-solving machine called *algebra* and see how easily, how accurately, how almost magically this situation is analyzed and the correct solution revealed.

Let x represent the time flying out
then $4-x$ = the time back
$150x$ = the distance out
and $100(4-x)$ = the distance back
then $150x = 100(4-x)$, since these two distances must be equal.

Now if we set in motion more of the machinery of algebra to solve this equation we have the answer to the problem. Note please that all thought of airplanes is now dropped for the moment. The laws of algebra, the operations utilized are so powerful, so general, so universal in their application that the tentative meanings assigned the symbols are of no consequence. As far as algebra is concerned the symbols could stand for miles, pounds, hours, dollars, yards, ages, coins, or any of a multitude of possible units. The material in the problem has been put into an algebraic equation, and, just as the raw materials are hauled into one end of the River Rouge plant and finished automobiles and airplanes come out the other end, so here the problem materials are fed into an equation whose solution is the answer to the problem. Thus,

$$150x = 100(4-x) = 400-100x$$
$$250x = 400$$
$$x = 1.6$$

Therefore the time going out is 1.6 hr.
Check:
$1.6 \times 150 = 240$ miles, the distance out.
$240 \div 100 = 2.4$, the time to return is 2.4 hr.
1.6 hr. + 2.4 hr. = 4 hr., total time.

Apparently the pilot will be able to return under the conditions of the problem if he travels out for 1.6 hr., 1 h. 36 m., truly a remarkable solution revealed simply and quickly by the magic of algebra.

However, the real mathematician is not at all satisfied with such a solution. He wants a formula that will work for any rates and any time and also involve the least work with the least possible chance for error. This can be done by analysis or by more extensive use of letters.

First the 1.6 came from 400/250. The 400 came from 4 times 100 and the 250 from 100+150. This suggests that the time going out equals the product of the total time by the rate back divided by the sum of the rate out and the rate back.

$$\text{Time out} = \frac{\text{total time times rate back}}{\text{rate out plus rate back}}.$$

This analysis is accurate but not so satisfying as the general solution which follows.

Suppose t represents the total time, r_o the rate out, r_b the rate back.
Then

$$r_o x = r_b(t - x) \quad 1$$

$$r_o x = r_b t - r_b x \quad 2$$

$$x(r_o + r_b) = r_b t \quad 3$$

$$x = \frac{r_b t}{r_o + r_b} = \frac{r_b}{r_o + r_b} t, \quad \text{or} \quad t_o = \frac{r_b}{r_o + r_b} t \quad 4$$

Expressed as a rule this becomes, *Time flying out equals the total time multiplied by the fraction, rate out over the sum of the rates out and back.* Furthermore, since distance = rate times time, r_o is the rate out, and t_o is the time out, then

$$D = r_o t_o \quad 5$$

$$D = \frac{r_o r_b}{r_o + r_b} t \quad \text{from (4) and (5)} \quad 6$$

Numbers 4 and 6 are truly marvelous equations; remarkable formulas. They came simply, easily, accurately by applying some of the simple universal laws and techniques of algebra. Again I say algebra is an ingenious invention of the human mind, a machine for solving elusive quantitative problems. As such it is more important than airplanes, automobiles, radio, or any other physical invention because in most cases these physical marvels could never have existed without algebra. As teachers of mathematics let us not lose our vision of the real significance of algebra, let us not get so engrossed in teaching techniques that we forget major values, let us not fail to see the trees for focusing on the leaves, nor the forest by looking only at the trees.

Suppose we do a little more analysis of this situation using the following symbols:
T = total time, t_o = time out, t_b = time back.
R = radius of action or distance out.
r_o = rate out and r_b = rate back.
Then

$$T = t_o + t_b \quad 7$$

$$R = r_o t_o, \quad \text{or} \quad R = r_b t_b \quad 8$$

Therefore

$$r_o t_o = r_b t_b \quad 9$$

or dividing 9 by $r_b t_o$

$$\frac{r_o}{r_b} = \frac{t_b}{t_o} \quad 10$$

Stated in words equation 10 reveals that the rates out and back are inversely proportional to the periods of time out and back, or that the faster you go out the less time you can use if you wish to return safely. These statements make sense and are revealed concisely by equation 10. Equation 11 is formed by adding 1 to both sides of equation 10.

$$\frac{r_o}{r_b} + 1 = \frac{t_b}{t_o} + 1 \quad 11$$

$$\frac{r_o + r_b}{r_b} = \frac{t_b + t_o}{t_o} = \frac{T}{t_o} \quad 12$$

Then

$$t_o(r_o + r_b) = T\, r_b \quad 13$$

$$t_o = \frac{T\, r_b}{r_o + r_b} = \frac{r_b}{r_o + r_b} T \quad 14$$

Formula or equation number 14 is the same one we got before by means of a strictly different approach, number 4. It reveals the consistency of algebra as well as its versatility.

The solution of the problem just given has been used for two purposes: first, to illustrate the power of algebra, its vitally practical applications, its universal nature, its machine-like qualities; secondly, to il-

lustrate the relationship between equations and formulas and the dependence of both on algebraic techniques such as factoring and using fractions. The equation is used to solve a problem, a formula provides a general solution. The latter is the efficiency expert of our modern scientific civilization. In it there is least chance for error, least work to get the answer, and least time consumed. The derivation of a formula should be one of the major goals of algebra teaching. No student has acquired the most significant values of algebra for war or peace unless he has developed the ability to make formulas for solving problems.

Perhaps a few more problems will help to illustrate further these two ideas and to establish them more firmly:—one problem from algebra not commonly generalized into a formula, one from geometry which could be made into a formula, and the third a business problem which can be basically algebraic and is greatly simplified thereby. The second problem has been purposely selected as one that borders on the artificial.

A city such as Oxford, Ohio, is supplied with water by means of several pumps which fill a tank about 100 ft. high. One pump could fill the tank in 12 hr. if the water level held up. Another can fill it in 10 hr. Suppose both were turned on, how long would it take, assuming that the water in each holds out and for convenience that no water is used during the interval?

Problems like this can border on the unreal and are the source of much ridicule for algebra texts and problems. You will recall the answer given by a bright boy to a problem about mowing a field for hay. One man could do it in 6 hr., another in 5 hr., and a third in 4 hr. How long will it take the three working together? The bright answer:— "Zero hours, the field having been mowed three times already, it would be futile to try for a fourth crop." Or, an aviator was asked, "If one plane can travel from Toledo to Oxford in 2 hr. and another in $1\frac{1}{2}$ hr., how long will it take both planes to reach Oxford?"

Note that the typical work problem as I stated it avoids artificiality and does become a real problem. It is needless for me to call attention to the futile ways of dealing with this situation. Some students will add the hours, some will average the hours, some take half the average. The solution follows:

Let

x = the time in hours for the 2 together.

$\frac{1}{x}$ = the part filled in 1 hr. by the 2 pumps together

$\frac{1}{12}$ = the part filled in 1 hr. by the first

$\frac{1}{10}$ = the part filled in 1 hr. by the second

Thus

$$\frac{1}{10}+\frac{1}{12}=\frac{1}{x}$$

$$12x+10x=120$$

$$22x=120$$

$$x=5\tfrac{5}{11} \text{ hr. or 5 hr. 27 min.}$$

The answer 5 hr. 27 min. solves the problem but it reveals nothing about the important functional dependence of the answer upon the original data. Suppose we try for a formula, first by analysis, then by more rigorous methods.

Where did the $5\frac{5}{11}$ come from? $\left(\frac{120}{22}\right)$

Where did the 120 come from? (10 times 12). Where did 22 come from? (10 plus 12.)

The formula by analysis is $t=\dfrac{10\times 12}{10+12}$, or, if t_1 and t_2 represent the times for 2 pipes, then $t=\dfrac{t_1 t_2}{t_1+t_2}$.

More rigorously determined, using the same letters, the solution is:

Let t = the time for the 2 pipes. Then

$\frac{1}{t}$ = the part filled by 2 together in 1 hr.

$$\frac{1}{t_1}+\frac{1}{t_2}=\frac{1}{t} \qquad 1$$

$$tt_2+tt_1=t_1t_2 \qquad 2 \text{ clearing of fractions}$$

$$t=\frac{t_1t_2}{t_1+t_2} \qquad 3 \text{ factoring and dividing}$$

The formula reveals the rather complex relationship involved in what seems a simple problem. It is the efficiency solution for problems of this kind. It could be used for mass production or mass solution of such problems. It is the finished product for computational safety and economy in 1943 both in war or in peace.

Suppose I plan to fly back to Oxford this afternoon. Several nice problems in geometry would be involved, only one of which I shall present here.

From an aeronautical chart I found that a line from Toledo to Oxford crosses the mid-meridian at an angle of 205°. Suppose my plane travels, not too fast over our beautiful landscape, only 80 mi. per hour. There is a fresh to strong wind, using Beauford scale terminology, of 30 mi. per hour from 150°. What correction in heading must I make for the wind?

The navigator measures angles *clockwise* and from the North. The line EX (Figure 1) represents the *true course* of the plane from Toledo (E) toward Oxford. EW represents in direction the *direction of the wind* (from 150°) and in length to scale the *speed of the wind* (30 mph). When a line segment is used in this way it is called a *vector*. EW is the *wind vector*. Since the airspeed is 80 mph, using 80 mi. to scale as a radius and with W as a center draw an arc cutting line EX in point P. Draw WP. Through E draw EH parallel to WP. Triangle EWP is a vector triangle in which EW was made the wind velocity, WP is the airspeed of the plane and the direction for heading the plane, EP is the actual or ground speed of the plane and the actual course of the plane. Each line represents two quantities, a direction and a speed. The angle EPW (18°) or its equal PEH is the angle to head into the wind, called the wind correction angle or the predicted drift angle. EH or WP is the direction for heading the plane. (205° − 18° = 187°.)

Solution—Wind Problem

$a = 80$ mph

$w = 30$ mph from 150°

$T.C. = 205°$ Toledo-Oxford

$W.C.A.$ (predicted drift angle) $= 18°L$

$T.H. = 187°$ (205° − 18°)

$g.s. = 60$ mph

$D = 160$ mi.

$$T = \frac{160}{60} = 2 \text{ hr. } 40 \text{ min.}$$

Every geometry teacher should teach this problem or one like it. If mathematics is to function in the lives of those who learn it, its teachers must ever be pioneers on the watch for new applications, new emphases, new usefulness. This problem uses the navigators' measurement of angles, scale drawing, and an appreciation of the fundamentals of vector addition. To arrive in Oxford I would need to head my plane 187° and maintain a true airspeed of 80 mph. Then, neglecting for the moment other compass corrections, if the wind stays 30 mph from 150°, my ground speed will be 60 mph and my "track" 205°.

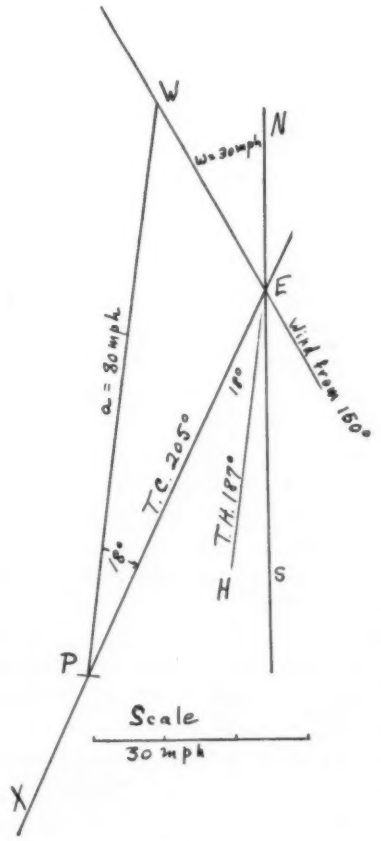

FIG. 1

For a final problem I have selected one from Social Security as follows:

Suppose a man wishes to compute his benefit for retirement in 1950, the law will then have been in effect 13 yr., and suppose his average salary is $200 per month in covered employment for the 13 yr.

Solution by Social Security Board:

40% of first \$50	\$20.00
10% of remaining average salary	\$15.00
Primary Benefit	\$35.00
1% for P.B. for each yr. 13% of 35 =	4.55
Total Benefit	\$39.55

Solution using algebra to get a formula:

Let A = average salary

n = number of years in covered employment

Then

$$40\% \text{ of first } 50 = \$20.00$$
$$10\% \text{ of } (A-50) = .1A-5$$
$$\text{Primary Benefit} = .1A+\$15.$$

Note already the saving in time and chance for error, if only the formula, $P.B. = .10A+\$15$ were used. Applied above it would be \$15 plus 10% of \$200, which makes the \$35 primary benefit.

$$1\% \text{ for each of } n \text{ years} = .01n(.1A+15)$$
$$= .001nA+.15n$$
$$\text{Total Benefit} = .1A+15+.001nA+.15n$$
$$= .1A+.001nA+15+.15n$$
$$= .001A(100+n)+.15(100+n)$$
$$= (.001A+.15)(100+n)$$
$$= .001(A+150)(n+100)$$

This formula, $T.B. = .001(A+150)(n+100)$, stated in a single simple rule would direct a computer of the monthly social security benefit under the present law to (1) add \$150 to the average salary, (2) add 100 years to the number of years in covered employment, (3) multiply these two quantities together and point off 3 places and the answer is dollars per month under social security.

Check:

$A = 200$, $n = 13$ yr. in the above problem

$$.001(200+150)(100+13) = .001(350)(113)$$
$$= \$39.55.$$

Note here that the machinery of algebra has short-cut several processes and steps in computation, has even concealed in its manipulations the complex relationships involved between total benefit and the two variables upon which it depends, and has revealed a very simple technique for the computation of social security. It is even conceivable that the use of this formula could relieve some congestion in Washington or at least reduce the number of possible errors in Social Security Computations.

Algebra makes a spectacular contribution to the solution and the analysis of a large variety of quantitative problems. The equation and formula are twin motors for one of the most remarkable machines ever invented, algebraic analysis. When these two motors are supplemented by another pair of twin motors also from algebra, the graph and the table, then one has a four-motored machine that can solve many complex, elusive problems, can contribute much to one's understanding of modern civilization, and will equip one to take an intelligent part in this age of science.

Teachers of mathematics who wish their students to be well prepared to live happy and effective lives in this scientific age must do two things. First, they must retain their vision of the nature and major functions of the remarkable invention called mathematics. Second, they must realize that no forest could exist without trees, nor trees without leaves, yet one can be so concerned with leaves as not to see the forest. Similarly, algebra and geometry can not exist without a multitude of minor techniques, principles, facts, definitions, and other details, yet one can get so engrossed in teaching algebraic and geometric techniques that he fails to emphasize major values. As teachers we must keep our mathematical perspective as well as our enthusiasm.

Education in a Time of Crisis

AT THE time of the first World War our schools were in grave danger. The average annual salary of teachers stood at \$655. Living costs rose sharply. Teachers left the schools. Many schools were closed. Delinquency increased. Our Association met these conditions by a great forward movement. Due in no small measure to the activity of the NEA, our schools made great progress following the war. Teachers' salaries more than doubled. Standards of preparation for entrance to the profession rose sharply. High school enrolment doubled and redoubled. Curriculums provided for individual differences. Now again we are faced with the necessity for action. Our profession must again meet the challenge presented by war conditions!—*George D. Strayer*, chairman, NEA War and Peace Fund Committee.

Conflicting Assumptions

By Earl Murray
Alturas, California

In many situations, good logical thinkers arrive at conflicting conclusions. Otherwise there would not be so many controversial issues. The social studies and business fields present us with many illustrations of this. Can a course in geometry, designed to teach the processes of logical thinking and nature of proof be of any value in understanding this situation? This is the report of a unit of work designed for that purpose.

It is a well known fact that assumptions are necessary and important in good thinking. In spite of this we have experienced many heated arguments in which very important assumptions were made but not stated. It is much easier to follow the thinking of an individual who is willing to give reasons for his opinions and who will state the assumptions upon which his conclusions are based. Many differences of opinion between individuals could be more easily adjusted if each would state and analyze the assumptions used, for if we think accurately and arrive at conflicting conclusions then we must have started with conflicting assumptions. It is obvious that a person who assumes that it is all right to break a traffic regulation if he is not being watched by an officer will exhibit a different behavior under certain situations than one who assumes that traffic regulations should be observed whether or not an enforcement officer is near. Our behavior is based upon our assumptions of conduct. A change in our assumptions will cause a corresponding change in our behavior and a change in behavior should be accompanied by corresponding changes in assumptions of conduct. When a person realizes that he is changing his behavior he needs to examine his former assumptions to keep them in uniformity with his actions. If his changing actions are out of harmony with his professed assumptions, then we have the individual who is inconsistent, emotionally upset, and who is confused in his thinking. Many poorly adjusted people are of this type, still trying to hang on to outmoded assumptions but not willing to act in harmony with them. We need to be honest with ourselves and realize that if we wish to change our actions, then a corresponding change in our assumptions must be accepted.

This particular phase of thinking which is so essential to life adjustment is very likely to be neglected in a geometry course because we have thought of our assumptions as being self-evident truths which were unchangeable. But if the main purpose of our teaching is to develop thinking power, then we cannot neglect this important phase of thinking. For many years we assumed a transfer value for geometry, which observation seemed to deny, and only recently have we been willing to accept the assumption that if we were to get transfer value from geometry, we must teach specifically for that transfer. The sad part of it is, however, that although we give lip service to this last assumption, our teaching has not changed to harmonize with it. This unit on "Conflicting Assumptions" is an attempt to teach something which will be functional in the life of the students.

In line with the thinking of the national committees on science and on mathematics the four basic aspects of living—personal living, personal-social living, social civic living, and economic living—were used as our areas for the study of conflicting assumptions and corresponding conflicting behavior. In addition to these, conflicting assumptions in geometry were also studied.

Conflicting Assumptions in Geometry

During the first day of the discussion of this new unit, the history of the study of parallel lines was presented with particular emphasis upon the discussion about the parallel postulate. It was recalled that for centuries mathematicians attempted to prove the parallel postulate; then, for other centuries they attempted to disprove it; and that it was not until a century ago that mathematicians came to view the parallel postulate as a simple assumption. Following this viewpoint, it was inevitable that they would take other assumptions about parallels; therefore, Riemann assumed that through a given point no line could be drawn parallel to a given line. Almost simultaneously Lobachevsky assumed that through a given point two lines could be drawn parallel to a given line. It is obvious that these conflicting assumptions would lead to conflicting conclusions. For instance, in Euclidean Geometry the sum of the angles of a triangle always equals 180°, while in Riemannian Geometry the sum of the angles is always greater than 180°, and in Lobachevskian Geometry the sum of the angles of a triangle is always less than 180°. It is interesting to note that many of Einstein's conclusions are derived through the Riemannian postulate; hence, we have such modern conceptions as there are no parallel lines, there are no straight lines, all lines are curved lines, and all surfaces are curved surfaces. This Riemannian assumption may be nearer the truth as far as the universe is concerned than is the Euclidean assumption, but for comparatively small areas of the earth's surface, we have a great need for the Euclidean assumption. As an illustration, we have plane surveying for small areas and geodetic surveying for measurement of large surfaces.

These conclusions and facts are interesting to us from the standpoint of our previous knowledge and training, yet, the greatest outgrowth of this study is to notice that changing our assumption leads us to entirely different conclusions even though no error is made in our thinking. In other words, our conclusions are dependent upon our assumptions and no conclusion can be any more reliable that the assumption upon which it is based.

Problems in the Basic Aspects of Living

The following outline was made as a cooperative result of teacher and class, with the teacher suggesting the main topical headings.

I. Personal Living
   1. Health habits—eating, sleeping, bathing, etc.
   2. Budgeting of time—study, work, recreation, rest, hobby, etc.
   3. Character traits—honesty, dependability, popularity, etc.

II. Personal-Social Living
   1. Home relationships—student and parent, brother and sister, etc.
   2. School activities—teacher and student, boys and girls, school and student body regulations, voting money, school awards, etc.

III. Social Civic Living
   1. Operating an automobile—safety, law observance and enforcement, etc.
   2. Governmental activities—protection of individual rights, administration of relief, public utilities, etc.
   3. Community activities—clubs, lodges, churches, community betterment, etc.

IV. Economic Living
   1. Budgeting of money—student, family, taxes, etc.
   2. Gambling—personal and social implications
   3. Advertising and merchandising
   4. Vocations—personal satisfactions and social implications

Procedures and Activities

The first activity was to get students to analyze reasons (accepted assumptions) and their logical conclusions in common situations of school conduct. One of the things which the class noticed in school conduct is a growing tendency on the part

of the students to run in the halls. Each student was asked to write what he believed would be the thinking of a student who finds himself in the position that if he ran he would be on time to class, whereas, if he refrained from running he would be tardy. These students wrote two paragraphs: one stating the process of thinking that a student would go through if he decided to run, and secondly, the process of thinking involved in case he decided not to run. These papers were discussed in class and from them lists of conflicting assumptions of conduct were made which were consistent with the particular act. It was pointed out that these assumptions have been called a variety of names, such as attitudes, moral standards, or ideals. The following are some of the assumptions listed:

TO RUN

1. It is all right to run in the hall if I am reasonably certain that it is safe.
2. It is all right to run if I don't get caught.
3. It is better to be prompt than to obey minor school regulations.
4. If I get caught, I might be able to talk myself out of the natural consequences.

TO WALK

1. It is never absolutely safe to run.
2. If I am late, I might be able to talk myself out of the natural consequences.
3. I want to be a good citizen, and good citizens obey school regulations.
4. It is more dignified and less rowdy to walk.

Following this the students were asked to reproduce the thinking of an individual while driving who would break the law because he was in a hurry if he knew no policeman was in the vicinity to stop him; and also to present the argument of an individual who did not break the law, even though he was in a hurry. A very short discussion of this disclosed the need for an agreement on the term "break the law." Some members of the class argued that breaking the law occurred when there was any infringement of the written code. Others argued that enforcement regulations were what really constituted the law. That is, the class realized that there is a gap between enforcement regulations and legislative acts; that many laws are outmoded, and never enforced, even though they still stand on the statute books. They pointed out that while the written law says 15 miles per hour across intersections and 25 miles per hour on our boulevards within the city, that a person could drive 30 miles per hour on these boulevards without fear of arrest. The class agreed that no one obeyed the written law, and that we would accept the regulations of enforcement officers as our definition of the term "break the law."

We tabulated the results of this thinking into fifteen assumptions for each situation. They are as follows:

A. ASSUMPTIONS USED IN ORDER TO DECIDE TO OBEY THE LAW IN DRIVING A CAR

1. Speed limits and traffic laws are passed for a reason, even if I do not happen to understand what that reason is.
2. These laws are established for the good of society as a whole.
3. I should put up with a personal temporary inconvenience to myself for the good of society.
4. Breaking the law influences others to do likewise and many people would cause accidents if they broke the law.

B. ASSUMPTIONS USED IN ORDER TO DECIDE TO BREAK THE LAW IN DRIVING A CAR

1. An emergency justifies breaking the law.
2. If there is practically no traffic, I am justified in breaking the law, as the law is made strict enough to take care of bad traffic conditions.
3. If I can see no reason for obeying the law, then I do not feel obliged to obey it.
4. It is my business if I want to go fast, I alone should decide how fast I should go.

5. Breaking the traffic law is dangerous since there are so many unforeseen ways that accidents can happen, such as blowouts, blind corners, skidding, etc.	5. The bad thing about breaking the law is getting caught; therefore, if I am clever enough not to get caught it is all right.
6. Higher speed increases the liability of accident and also the seriousness of the accident if there is one.	6. We must, of course, have drastic traffic laws for the average or poor driver, but these laws are not made for a superior driver like me, and so it is all right if I break the law.
7. Even a good driver can get into a wreck, which may be caused by the actions of a poor driver.	7. It will not make any difference if I go a little faster or break the law a little.
8. In case I should ever be involved in an accident while driving, it would be better for me if I were driving within the law.	8. If I have an accident it will be the other fellow's fault, not mine, since I am careful.
9. Better be safe than sorry.	9. I will look out for myself on the highway, and it is up to the other fellow to look out for himself.
10. A good citizen obeys the law.	10. We all want thrills, and it is thrilling to drive fast.
11. Haste makes waste.	11. It is fun to rebel against a law and feel that you got by with it.
12. There is a personal satisfaction in being a law-abiding citizen.	12. Breaking the law and bragging about it makes me feel like a "Big Shot."
13. I believe in the principle of law observance.	13. I will be better thought of by people if they see I am brave and bold enough to break the law. It satisfies my desire to show off—to get attention for myself.
14. I would be ashamed of myself if I needed an officer around all the time to force me to obey the law.	14. I feel under no obligation to obey the law if there is not an enforcement officer around. It is his business to enforce the law, not mine to obey it.
15. I want the reputation of being a law-abiding citizen.	15. I do not believe in law observance, but just in law enforcement.

After tabulating these assumptions the question arose as to what a person does who is conscious of the role of conflicting assumptions? The following statements describe the behavior of such a person.

1. He interprets behavior in terms of implied or expressed accepted assumptions.
2. He sees the need for change of behavior upon acceptance of new assumptions.
3. His behavior is consistent with his accepted assumptions.
4. He can be emotional and yet logical.
5. He is tolerant of the assumptions and resulting behavior of others even when in conflict with his own assumptions and behavior.

With these objectives in mind several exercises such as the following were given:

1. Situation: Suppose you drive up to a boulevard stop sign, you slow down and look around, you see no traffic which you believe warrants a complete stop, you see no officer around so you barge through the sign without a semblance of stopping.

Directions: Rate the thirty assumptions listed about obeying or breaking a law as to those which you most probably accept, those you most probably reject, and those which seem irrelevant to the situation.

Many problems similar to this can be made by altering the situation.

2. Write about an incident bringing in a situation involving the probable actions of a person who believed in any particular given group of assumptions picked from the list of thirty.

Many problems can be made by changing the given group of assumptions.

Conflicting Assumptions in Government

It is recognized that our differences of opinion concerning the functions of government are not due entirely to prejudice or lack of thinking. Many good thinkers differ as to the type of action advisable in performing certain functions of government.

We examined the written assumptions of our American government as expressed in the Declaration of Independence and the Constitution of the United States and expressions of Americanism as expressed by Presidents and others, such as Washington's Farewell Address, Lincoln's Gettysburg Address, The Monroe Doctrine, and writings of textbooks in Civics. A few of our national congressional acts were examined for the purpose of deciding which of the discovered assumptions were the basic ones used for the establishment of such an act. Since many of these acts are conflicting, we wondered if they were founded on conflicting assumptions. A re-examination of these assumptions showed that those stated in the Declaration of Independence and the first ten amendments to the Constitution leaned more toward the statement of individual rights, while those expressed in the Constitution stressed the general welfare. It was agreed that many of our recent governmental projects leaned more heavily towards promotion of the general welfare than toward some of the other assumptions. It was noted that the constant battle between the personal rise of the individual such as might be expressed in the term of "Rugged Individualism," and the development of the general welfare is a matter of stressing the assumptions in our interpretation of the functions and duties of government. It was hoped in this discussion that the student would gain a better understanding of the natural conflict between these two ideas and would be less prejudiced towards any opinion which was different from his own. That is, we hoped that toleration for the other person's viewpoint and better thinking on the part of each individual would replace traditional prejudice and emotionalism in political and social issues.

Assumptions in Advertising

It seemed advisable at this time to discuss assumptions in advertising, although, at first thought this does not seem to fit in very well with the unit on conflicting assumptions. Yet, from the standpoint of the effect of thinking about advertising, it seems to work very well.

Each member of the class was asked to bring in a number of advertisements, analyzing each one from the standpoint of expressing the assumptions which were the appeals in the advertisement. At the same time each student listed the assumptions which he would use as sales resistance to the ad. We discussed the validity of these assumptions, frequently finding conflicting or irrelevant assumptions which lead us quickly into the field of propaganda and rationalization. We were impressed by the lack of thinking in advertisements and by the appeal to the very primitive and elemental emotions. Our aim was to develop in the minds of the students a greater critical analysis of advertising.

Evaluation

A unit of this type is difficult to evaluate—either from the standpoint of proper student understanding of the relationship between assumption and conclusion or as to whether or not this type of activity contributes greatly to the broader and more general aims of geometry teaching. A test was given for the purpose of checking these student understandings and is explained by the following:

General Directions

This is a test to determine how well you can select the underlying assumptions which are consistent with resulting behavior. You are given a situation in which a number of assumptions may apply and you are asked to denote the assumption you would *most probably accept* and the assumption you *would most probably reject*, dependent upon the course of action following a description of the situation. Each course of action is followed by two squares. In the first square you are to place the number of the assumption that you would *most probably accept*, if you had behaved as described in the course of action, and place in the second square the number of the assumption that you would *most probably reject*, if you had behaved as described in the course of action.

Problem I was in law observance in which nine assumptions were stated, fol-

lowed by a situation with four possible courses of action. As per directions the students noted the assumptions consistent with and those opposed to each course of action. Problem II dealt similarly with a problem in choice of foods for lunch in a student cafeteria line. Problem III asked the student to list assumptions used in support of a Scholarship Society and those assumptions used for abolishing the Society. If there were conflicting assumptions in these two columns, the student was to note the fact and also state whether or not there were true assumptions in both lists. Problem IV asked the student to list the basic assumptions used by an ad in order to persuade him to buy, and then list those he would use as sales resistance.

This test was difficult to score but was probably as reliable as most ordinary teacher made tests. At least the class seemed to think so.

Each student kept a note book during the unit in which he placed his individual work and wrote up the class or group work. He was graded on this note book as well as on the test. He was encouraged to use the same care as he would on an English note book, and many fine note books were turned in.

In order to evaluate the general contribution of this, it was desirable to get student opinions regarding the worth of the unit expressed in such a manner that their grades would not be influenced by their statements. Students were asked, therefore, not to place their names on their evaluation sheet. It was also necessary that questions be stated in such a way as to allow adequate expression of degree of value, if any worth-while conclusions were to be obtained. With these criteria in mind, the following sheet was prepared and checked as follows by the students:

GEOMETRY EVALUATION OF A UNIT OF WORK ON CONFLICTING ASSUMPTIONS

Directions: In front of each question, write the number 4, 3, 2, or 1 which seems to you to most nearly answer it. 4 means yes, 3 means probably, 2 means probably not, and 1 means no.

Class Average on Answers		
3.35	1.	Will this help you to seek reasons for your actions?
3.35	2.	Are you more apt to question your actions?
3.41	3.	Will it help you to make your actions more consistent with your thinking?
2.79	4.	Will you have more respect for monitor and school regulations?
3.35	5.	Will you be a more careful driver?
3.28	6.	Will you have more respect for law?
3.35	7.	Has this given you a better understanding of the conflict in American government?
3.28	8.	Will you be able to analyze advertisements better?
2.86	9.	Will you be more tolerant of the thinking of others in politics?
3.52	10.	Will you be apt to apply this type of thinking to other problems which we have not mentioned at all?
3.20	11.	Will you look for definitions and assumptions in speeches and written articles?
3.41	12.	Will you try to be more careful to define your terms and state your assumptions in an argument?
3.52	13.	Do you consider the time spent on this as worth while?
3.41	14.	Do you think this type of study will do you more good than spending all your time on book geometry?
3.28	15.	Do you think this study will help you to realize the importance of assumptions in our geometry proofs?
3.29		Class average on each question.

ANALYSIS OF EVALUATION TEST

The general average of 3.29 shows a strong tendency toward improvement in both thinking and action as a result of this study. In general the first six questions deal with conflicting assumptions in conduct, and shows a decided improvement in coordinating thinking and action, with less improvement where force is involved, that is, in school regulations and general law. This bears out the argument that force is a poor tool to use in a learning

situation. The seventh and ninth questions indicate that a better understanding had failed to develop tolerance to any great extent in the field of politics. As one student expressed it, "it just gave me more evidence to support my political thinking, and more ability to attack the other person's viewpoint." We hope that further work in the social studies will do much to change this viewpoint. The eighth question indicated satisfactory results were obtained in the field of advertising, although very little time was spent on it. Questions ten to fourteen were an attempt to evaluate the general results as a unit, rather than the specific things studied. The average for these is higher than the average for the whole. However, there is much variation of response within this group. It is worthy of note that the questions which were scored the highest in the evaluation were the ones concerning the transfer value of this to other problems which were not mentioned at all, and number 13 which deals with the student's view upon the worth of the project. This would support the modern viewpoint on the transfer value of geometry teaching, namely that there can be transfer of training where teaching is set up to accomplish that purpose. This is a very fine illustration of the need for broadening our materials, as well as methods, if we are to achieve the important aims of geometry teaching. If your aims are still focused primarily on the acquisition of skills which can be measured by ability to give geometric proof, let us notice that the students looked quite favorably upon the last question, which stated that this study helped in realizing the importance of assumption in our geometry proofs.

Students were encouraged to write their opinions or criticism of this work on the back of the evaluation sheet. Many did this. A few students who were very fond of geometry resented the time spent, but even they said that it was helpful and worth while, but that it should be done in an English class rather than taking valuable time from a geometry class. The majority of the class were more enthusisatic. Here are two typical responses.

"I like this work and I hope we can have more of it in the future. It really stimulates thinking, and helps a person, I believe, in his other subjects." "I have enjoyed this work an awful lot. Now I look before I leap."

The students were encouraged to take their parents into their confidence about this work, and many of them did so. Each student was allowed to take one of the tests home to his parents. Many of the parents spoke favorably to me about the project—and none spoke unfavorably, or objected to the amount of time spent upon it.

THE enrollment in the public high schools for the year 1943–44 is 5,761,000, or about one million below the peak enrollment of 6,714,000 in 1940–41, a preliminary estimate made by the U. S. Office of Education of the Federal Security Agency indicates, the FSA said on Dec. 9, 1943.

The estimated present enrollment, made up of 2,701,000 boys and 3,060,000 girls, is approximately the same as the total in 1933–34. The 1943–44 enrollment is 6.2% less than last year, and represents a drop of 246,000, or 8.3%, among the boys and 135,000, or 4.2%, among the girls.

Enrollments in the junior and senior classes of high schools have declined between 9 and 10% since last year. The nun ber of boys declined about 15%, the number of girls about 5%. This drop is probably accounted for by the large numbers of students who have left school for work in industry and for service in the armed forces.

The importance of completing their training is being continuously urged upon young people by Paul V. McNutt—who is concerned in the school-and-work problem both as Federal Security Administrator and as Chairman of the War Manpower Commission—by John W. Studebaker, U. S. Commissioner of Education, and by Katharine Lenroot, Chief of the Children's Bureau, U. S. Department of Labor.

How Shall Geometry Be Taught?

By M. Van Waynen
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Recent investigations which have compared the merits of the traditional geometry course and the newly inspired course which focuses attention on the principles of logical thought and its application to everyday situations have strongly favored the latter. Results of these studies indicate that pupils of the experimental classes acquire the usually attained geometric achievement while definitely managing to surpass the control classes with scores of nearly two to one in reasoning achievement. However, in spite of these highly satisfactory results, most high schools have done nothing toward adopting the newer techniques which have made them possible.

This hesitancy is probably due to the confusion which exists regarding the contribution of geometry to a pupil's education. It seems safe now, however, to postulate, with ample support of many leading mathematicians, that the aim of geometry is the development of the pupil's ability to reason logically in all fields of thought.

Teachers, who in general agree that geometry can be taught so that it will develop these powers of reasoning more readily than other subjects, admit that geometry is not ordinarily taught in such a manner. Most instructors still see no reason for the actual application of geometry to unrelated situations and, as a consequence, have gone little beyond the enrichment stage.

This attitude probably rests on the assumption that the logic of geometry would automatically be transferred to nongeometric fields. The few scattered investigations which have substantiated this have, however, been quite effectively discredited by the numerous studies made which show contrary results. These latter studies show quite vividly that the transfer of the thinking processes of geometry to unrelated fields of thought takes place only when it is learned in connection with the field to which transfer is desired. Teachers should no longer assume that the study of a few selected theorems and their associated applications will enable their pupils to think logically elsewhere than in geometry.

Recognition of the theories of transfer have gone little beyond the emphasis placed on the informational aspects of geometry to life activities. Even those who believe that this transfer can be attained by studying the processes themselves, with no need for making applications, must meet early defeat. Professors of education have not been kind to teachers of mathematics of late and have, naturally, stirred up antagonism to themselves and their dictums. Yet do not those who have the weight of evidence on their side have a right to strike hard?

Those who contend that geometry should be taught as a science dealing with the simple relations of space assume that geometry is an "applied mathematics." This assumption originates in the fact that geometry in its inception was a science devoted to the study of positional relations of rigid bodies. However, since that time, Euclid transformed it to the point where it became the ideal of deductive science and logical thought in general. The recent innovations in the teaching of geometry support this view, contending that one is more justified in teaching geometry as a course in "applying" mathematics, mathematics being in the final analysis the fundamental form of logical thought in any field.

The title "geometry" suggests the science of earth measure, while its place in the mathematics curriculum implies that the emphasis is on logic. Which side is right will depend on the results obtained. Teachers of geometry are on the verge of

making a momentous decision. Even the most timid must admit that the course which can impart the more essential facts of geometry while achieving transfer to everyday problems should have little to fear from even the most skeptical.

The outstanding discovery of the experimental studies has been that transfer to other fields can be accomplished easily. One might reasonably expect that a so-called scientific subject could supply an abundance of applications to show the importance of definitions, assumptions, and deductive reasoning. Few of us, however, realize how prolific are the opportunities for applications in our everyday reading. Too many teachers, who valiantly endeavor to impress Johnny and Mary with the importance of the basic elements of logical thought as used in scientific discourse, have failed to see its even greater importance in everyday discussions that affect life in so many of its vital phases.

Craftsmen are not likely to erect durable structures without giving considerable thought to their tools. Yet writers who study synonyms, derivations, rhythm, and style very rarely explore the nature of words themselves. They assume that the reader knows exactly what they mean. One could even go further and suspect them of using words which they know will be interpreted by the reader to satisfy the reader's own ego. It is comforting for the ordinary individual to find a famous writer agreeing with him!

Ask several of your friends to classify some prominent politician as conservative, liberal, or radical. Do not be surprised to find the politician "divided up" into some such percentage as 20% conservative, 40% liberal, and 40% radical. Go a step further and call for definitions of the above words, remembering that such words are commonly used in political controversy.

Should politicians be permitted the liberties of a poet? John Keats has said, "Beauty is truth, truth beauty"; our leaders should be more explicit. A case could be built up for the vagueness of the Atlantic Charter, for the necessity of political double talk during the stresses of war. Before the emotions of the world become too snarled perhaps we should define the peace aims in order to clarify our war policies.

Teachers in general and geometry teachers in particular should recall Alice's encounter with Humpty Dumpty on the other side of the looking glass.

"When I use a word," Humpty Dumpty said in rather a scornful tone, "it means just what I choose it to mean—neither more nor less."

"The question is," said Alice, "whether you can make words mean so many different things."

"The question is," said Humpty Dumpty, "which is to be master—that's all."

The necessity for recognizing basic assumptions is no less acute than that of establishing precise definitions. Good writers generally introduce their basic assumptions early in their articles. In this way the reader knows exactly where he stands and is in a position to follow through subsequent deduction.

Secretary of State Hull's advocacy of the Reciprocal Trade Agreements Act is based on his belief that "nations cannot sell without buying." It is this assumption that his opponents should assail if they wish to disprove his arguments logically. It is basic to the issue of Protectionism vs. Free Trade; a historical divider of American political opinion.

Someone has said that the reason for Felix Frankfurter's refusal to join political parties is based on his final and unalterable assumption about the nature of human-beings and the meaning of the world. The judge does not belong among the party members but belongs instead in the long honorable, but not recently honored, tradition of those who think of government not as a partisan's prerogative, but as a wise man's study. This assumption simply but clearly depicts the character of the man.

The Republican Program Committee of 1940 assailed the so-called New Deal leadership by saying that the New Deal began with the assumption that "the once expanding dynamic enterprise of America is maturing towards a static economy." If the New Deal did base its program on this assumption, then the Republican endeavor to show the assumption false was a logical step forward in convincing voters to vote Republican.

Lippmann's memorable article "The American Destiny" is based entirely on his effort to refute the assumption he pointed out:

> For the moment, this moment of bewildered indecision through which we are living, the American people are acting on the assumption, though in their own hearts they reject it, that their incomparable assets are in fact their most dangerous liabilities.

A circular from the National Council for the Prevention of War calls an assumption by the name of "axiom" in the following:

> It is an axiom among military men that the most difficult military operation is landing troops on hostile soil, and a country cannot be invaded without troops.

Other war axioms have cropped up in recent writing. One, that some of us forgot, was that today's war must be fought with today's equipment. Another states that each time an army advances it creates a future need for further advance to protect the newly acquired position. It was not easy for General McNear to bring the hard cruel facts of war to a generation reared on the premise that there would never be another war. The allies underestimated the Japanese, who refused to admit that there were natural obstacles their forces could not cross, because the allies believed that terrain which was merely difficult was impassable!

In the realm of education some of the existing laxity in maintaining scholastic standards can be attributed to a grading system based on normal curve distribution, which in turn rests on the assumption that the students being graded have cooperated to their fullest capacities.

Mr. J. Edgar Hoover of the F.B.I. has been condemned because he dares to base his reasoning on the assumption that crime rests on the doorstep of the American home. Have you ever known of a teacher who traced his special student peeves back to the same source?

Assumptions are rampant and it is not difficult to find them. They must be found because the settlement of any dispute must unquestionably lie in the choice of assumptions. Even though one does not know whether an assumption is true or not he must act as if it were true. To agree on assumptions is usually tantamount to agreement on conclusions. One must also be careful of misleading and omitted assumptions. Just as a chain is no stronger than its weakest link, so no proof is better than the assumption on which it is built.

A goodly portion of today's journalistic endeavor cannot be praised for lucidity of thought. The logical patterns around which an article should be written are generally obscure, if existent at all.

For this reason, locating articles written around a sound logical framework is by no means as easy as locating the ubiquitous assumptions. Perhaps this means that good writers are a rarity; perhaps too much is written for purely emotional appeal.

Even though one does not endorse the Ruml plan, teachers of mathematics must admire his reasoning. He reasons that 1943's aggregate income is bound to be more than 1942's. Even without any increase in tax rate, if this year's taxes are paid on this year's income the Treasury would be bound to collect more than if it were receiving payments on 1942 income —as it would be under the present tax law. Over the long term, national production and hence national income will probably increase. Under that assumption the happy situation outlined for 1943 will continue with cyclical variations. Come

Judgment Day, no one would owe the Treasury anything, whereas under the present system Judgment Day would find Mr. Morgenthau's successor with a year's unpaid taxes still due.

Another excellent example of deductive thought is furnished by Dr. Shushi Hsu, an outstanding Chinese citizen, who foresees the possibility of Chinese collapse. He points out that the Japanese blockade is affecting China just as blockade strangled the Southern States during the Civil War. China's needed supplies were mainly derived by smuggling through the coast. The conclusion is clear.

Rationing owes its existence to a great extent to violations of one of the oldest theorems of practical economics. This theorem states that if people have a lot of money, and if prices are held down, goods will go off the counter. The logic that created this simple theorem can easily be reconstructed.

But the best example of deduction is supplied by a pupil who, you will agree, derived something from geometry that many miss. In the following steps, ably substantiated by trustworthy references, he proved that the family is the most important unit in our democracy.

A. Personal character is a growth, the conditions of its nourishment being the social relations in which it is formed.
B. The closer and more intimate the group relationships and contact the more powerful the effect upon the growing personality.
C. The family is the most intimate and most closely united of any of the social units.
D. The family is the most influential social unit in the development of personal character.
E. The development of personal character is the most important function performed by any social unit in our democracy.
F. Therefore, the family is the most important social unit in our democracy.

An example of simple, indirect reasoning was furnished by an unnamed editor who was greatly mystified by an isolated sentence in a Roosevelt message in which he stated, "even today we are flying as much Lend-Lease material into China as ever traversed the Burma Road. . . ." If so, the editor reasoned, the number of U. S. transport planes sent to China's aid had been carefully concealed; or the amount of aid over the Burma Road had been smaller than anyone knew; or the President had based his statement on overoptimistic reports. Apparently the first two possibilities were eliminated because conditions in Chungking did not justify them.

One could list example after example of transfer that might have been inspired consciously or unconsciously by the logic of geometry. It certainly would be a feather in the caps of the teachers of mathematics if these obvious applications of logic in reporting could be attributed to time spent in a geometry class.

Buy War Bonds
and
Savings Stamps Now!

Readiness in Mathematics*

By CHARLES O. RICHTER
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THE TITLE of my talk this morning has a very ambitious sound, and I would prefer that it read "One Man's Approach to the Problem of Readiness for Mathematics in the Junior High School." The title on the program suggests that I have something new and startling to present and therefore if you came for that reason you will be greatly disappointed. There is very little new, in my opinion, in any branch of our educational system but frequently truths are rediscovered and interpreted differently by different people. Witness the rediscovery that there is great value in having children read orally. The practice used to be to have approximately 90% of all reading in the sixth grade silent reading. Hence my effort this morning will be concerned exclusively with one man's application of his understanding of child growth and development as it is applied to arithmetic.

It is a very rare occurrence indeed when a reformed social studies teacher gets an opportunity to talk to such a group as this. I think, however, that my having once taught social studies has given me a rather unique opportunity to notice the implications of arithmetic for classes outside the arithmetic period.

The demands of war on the field of mathematics has exceeded the supply of trained mathematicians just as the demand for skilled workers in other fields has exceeded the supply; welders, burners, draftsmen. This is also true in the matter of supplies for waging a total war. The demand for rubber, tin, and iron has far exceeded the supply. The way in which the people of this country have adapted their living to these shortages and have set about to remedy the situation is nothing short of remarkable, in spite of the mistakes which may have been made. In the same way that industry has set about training skilled workers, so have our schools attacked the problem of training people in skills in mathematics. It is my opinion that the job has been as well done in the schools as in industry; therefore I am not particularly disturbed about the mathematical skill of men and women needed in industry and in the armed forces. Some alarmists have hysterically pointed a shaking finger at what they call the deplorable training which the majority of our children are receiving in arithmetic. Moreover, these people feel that education, as far as arithmetic is concerned, is missing the boat. In reality, however, the situation in the field of mathematics was no worse than in any other field when war came two years ago. It is true that education was geared for a peacetime way of living and there was, perhaps, not the great need for the teaching of arithmetic as a science just as there was not the need for great quantities of war material, or great numbers of skilled workers. Had we not been able to meet the challenge of the war, education could undoubtedly have been held strictly accountable.

Criticism of the products of our educational system is not new and undoubtedly we have profited by it. Not only has the taxpayer and the employer made comments on our educational setup, but one level of the school system criticises the work of another level. As a teacher of social studies in the elementary schools, I grew quite accustomed to hearing junior high school teachers wonder what the elementary school taught anyway when John could not even locate Budapest on the map; or later as a teacher of Junior High School mathematics hearing the

* An address given at the December (1943) meeting of The New England Association of Mathematics Teachers.

Senior High School people say that there was nothing enough wrong with the elementary or Junior High School that a good Senior High School could not correct in time to prevent the taxpayer from discovering it. The teacher of physics rebels at the need for his teaching decimals to his physics classes. Truly I used to wonder myself as a teacher of high school Latin what the English teachers were doing about the putting across of such things as nouns, verbs, and the like. And so the story goes.

Why the charges and countercharges? Is the situation in arithmetic as serious as is claimed? Is it an indictment of our teaching of arithmetic because so many youngsters trained for peacetime world fail army examinations. It would seem that an understanding of how youngsters grow and develop should reassure teachers and parents concerning the difficulties which youngsters encounter in many fields of education. I believe we have let ourselves become agitated unduly about the situation. There is no research available which will support the statement that pupils today are more poorly prepared in arithmetic than they were when Dad was a youngster.

There are, however, more pupils in our schools who are weak in arithmetic than when Dad was a boy, but then there are more pupils in school today. There are several reasons why we have pupils in our schools who are weak in arithmetic. Among them are the following: First . . . about fifteen or twenty years ago only twenty or thirty per cent of all the children of secondary school age attended school. In other words we had more or less the cream of the milk bottle in school on this level. The picture has changed tremendously. Today, between seventy and ninety per cent of our youngsters of secondary school age are in school. Therefore it is natural that we should get down into the skim milk section of the milk bottle. In my opinion this accounts for a certain segment of our school population being less able to handle number concepts as well as concepts in other fields.

People forget—youngsters and adults alike, the things which they do not use. It is not particularly alarming to me then that an otherwise normal high school senior cannot demonstrate on the instant the division of fractions. In most cases this is because he has had little or no need for using division and therefore, temporarily, he is in difficulty. It may be that there is something wrong in the program of a school or in the life of a youngster when for several years he has not found it necessary to use the division of decimals. I would hope, however, that the school had influenced the youngster to the point where he would know where and how to get that information.

I was in a certain junior high school last week and the principal of the school told me the following story. One of the 7th grade pupils took the following problem home to her parents, "Find the product of .08 and $\frac{1}{2}$." She came back to the teacher on the Monday following and reported that her father said that the problem could not be done, that it did not make sense. He had worked on it for some time over an hour and could not do it, so the following morning he took it to his employer, a man successful in business to the extent of several million dollars, for him to try. This man toyed with the example for some time and asked to take it to a professor friend of his, who taught Spanish at a college near by. The professor could not do the example either and returned it saying that as far as he was concerned it was ridiculous. The indignant parent sent a note with the youngster to the teacher asking why his 7th grade daughter was getting work in mathematics which three adults could not fathom.

In the October 1941 issue of THE MATHEMATICS TEACHER, in an article entitled "What Price Enrichment" by J. O. Hassler, is found what on the surface appears to be a serious indictment of the teaching of mathematics in our public schools. The author of the article points

to a study carried on at the University of Oklahoma. Placement tests given to about 1300 freshmen uncovered the fact that 16% of the students could not multiply $.65 \times 32.4$; 11% could not add $2\frac{1}{3}+\frac{1}{7}$; 57% could not express $\frac{7}{8}$ as a decimal fraction. Certainly this is a challenge for the schools feeding into this University. One would hope, however, that the same significance is not attached to this report as was attached to the recent history test administered under the direction of the *New York Times* to college students. While some good may have resulted from this history test I believe it has been pretty well demonstrated that more importance was attached to the results of the test than was warranted. In the case of this arithmetic it would seem that the cause probably was this matter of forgetting through nonuse and that the remedy lies in remedial instruction in those secondary schools where there is nothing in the curriculum, which, from time to time, calls for the use of fundamental operations.

I dare say that there are people in this audience who can not tell us the capital of North Dakota, or the capital of Minnesota. We have had less emphasis on this kind of teaching for some time because it has been demonstrated over and over again that people forget what they do not find they need to use in their daily living. Forgetting, then, is one of the reasons why teachers get pupils in their classes who seem to be poorly prepared or to be slow-learning pupils. The efficient teacher is, of course, quick to spot this and to take steps that seem necessary in individual cases to remedy the situation.

I wish to quote the following from "The Implications of Research for the Classroom Teacher," a joint year book of the American Educational Research Association and the Department of Classroom teachers (see p. 202). "Some studies show a high degree of retention but most of them indicate very little. Nearly every algebra teacher has observed the low level of mastery of the fundamentals of arithmetic. Many studies of the retention of arithmetic by secondary-school pupils have been made, and in general, the results have been disappointing. Tests given to college freshmen have indicated a higher grade level in problem solving than in computation. In algebra, also, it has been found that there is more retention of those aspects of the subject which involve thinking than in those involving mechanical manipulation. There seems to be very little retention of the ability to draw logical conclusions in the field of geometry."

This is a problem which some schools are beginning to face. At the Newton High School this present week an arithmetic test was given to all high school seniors for the purpose of locating those pupils who cannot or have forgotten how to perform the fundamental operations; and who are not able to solve problems with which the average adult is faced. The students, so discovered, are to be given special teaching in arithmetic by the teachers of the mathematics department during certain periods of the week. This remedial work will be in addition to their regular course in mathematics. The students in Grade 10 will be tested shortly also and a similar program outlined for this group.

The third cause for lack of success in arithmetic, frankly, is probably due to poor teaching. Youngsters have had computation stressed at the expense of understanding and of problem solving. Teachers generally prefer drilling for the mastery or operative skills because the process is more easily measured and progress more discernible. Drill has preceded understanding and has therefore left youngsters without a mode of attack when new situations present themselves. Moreover, altogether too often, it happens that teachers are unwilling or unable to interpret the course of study to meet the needs and interests of the particular individuals whom she is charged with teaching, but year in and year out applies the course of study to the groups which come before her in the same way.

Too often arithmetic is taught as a separate subject entirely unrelated to the other school experiences of youngsters. I realize that I am treading on dangerous ground and that a storm of protest could easily be started on this subject. I am not advocating the teaching of the so-called "social arithmetic" exclusively, nor am I in accord with those who believe that mathematics should be taught to all pupils as pure science. It does not seem to me that an "either/or" choice needs to be made and the result be applied to all pupils generally. The truth probably lies somewhere between the two, depending upon the type of youngster with which we are working. However, it is true, it seems to me, that students often fail to see the relationship between a science problem involving arithmetical concepts or to grasp the arithmetical implication in map study in a social studies class. I do not mean by this that the teaching of arithmetic should be incidental to the teaching of social studies or science, but that the teacher should inform herself as to the other experiences in other subjects which children are receiving and that she should then modify the course of study and, wherever possible, use this background in other fields to make children more conscious of arithmetic as it concerns their total living. Moreover, it is a two way street. Teachers of science, of the social studies and other teachers should use as tools the number skills and concepts which their pupils are mastering during the arithmetic period.

It seems to me that the greatest problem with which all teachers must cope is that of lack of readiness on the part of students for the program of a given grade or level. It is my judgment that the greatest single cause of failure in mathematics is due to this immaturity of the youngsters, many of whom are not ready to cope with the number concepts required by the level on which their chronological age has placed them. The accumulation of difficulties from year to year usually results in a permanent disability in arithmetic and a dislike for the subject.

It seems to me that the older a youngster becomes the less recognition adults generally give to the factor of readiness. Those of you in this audience who are parents know that two children in the same family do not develop and mature at the same rate. The mother often comments on the fact that John is taking a longer time to walk than did his older sister Mary, that John has two teeth while Mary's first tooth did not appear until several months later. No one in the family seems particularly disturbed by any of this. A wise mother is perhaps the world's best teacher because she does not expect the same performance physically, mentally, socially, or emotionally from all her children. You all, perhaps, have witnessed the scene where little John for the first time struggles to his feet. Mother is delighted. She holds out her hands encouragingly. She does not force the youngster and soon John takes his first step. The youngster is not forced into walking before he is ready and yet mother is quick to see signs that John wants to walk and, wise teacher that she is, she provides every opportunity possible.

Somehow, however, a change takes place in the thinking of many parents just as soon as the children enter school. Because youngsters are in the first grade they are expected by the family and some teachers to begin reading at the same time. It is wrongly assumed by them that all the youngsters in any grade are ready for the learning of number concepts and for the same number experiences. Altogether too often this story is repeated in every grade throughout the youngster's school life. We do know that youngsters in the same class differ widely in their readiness for learning, and yet we use the same books, the same course of study and the same teaching procedures for all youngsters. A youngster in grade 7, for example, may be succeeding on a fourth grade level in reading and a 7th grade level in computational arithmetic. His writing may show

evidences of his physical immaturity to the level of a fourth grader, while in spelling may be equal to that of a sixth grade pupil.

Let us for example take three classes which I know about. These are actual classes meeting in a near-by junior high school. The 7th grade in that school is divided roughly into three large groups: Level 1 is composed of youngsters with high I.Q.'s; level 2 is made up of youngsters of average intelligence and level 3 is made up of slow learning youngsters. I should add that the bases for this division were two; intelligence rating plus teacher's judgment. Each of these levels in turn is divided into classes of about 35 each and on each level these classes would of course be of approximately the same ability so that we would have 7–1a and 7–1b composing the upper level, 7–2a, 7–2b and 7–2c composing the middle or average level and 7–3a and 7–3b making up the lower level.

The first class, which I am going to tell you about is 7–1a. Let us look at their scores in terms of grade equivalent made on a city-wide test when they were 6th graders. The highest score in arithmetic was 10.4, the lowest 6.2 giving a range of 4.2. In reading these were the grade equivalents, 10.5 for high score and 4.7 as the lowest score, giving a range of 5.8. Now let us turn to a class in the middle division. The high score in arithmetic 8.3, low score 6.1 and the range 2.2. In reading the high score in this class was 8.8 and the lowest score 3.8 making a range of 5 years. In one of the classes on the lower level the highest score was 7.4 and the lowest 4.4, the range 3 years. In the same group the highest score in reading was 6.2 and the lowest 3.4 giving a range of 2.8.

This story does not vary a great deal in this school from year to year. I know this because as a teacher in that school, I was faced with this kind of problem when I entered the service there. I was not long in discovering that there were certainly all degrees of readiness for the program of that 7th grade, and rather desperately I set about to interpret, by omissions and corrections, the course of study in mathematics grade 7.

It is my judgment that the situation described above is not peculiar to that particular school but that the description would fit many schools throughout the country.

Does this situation have implications for the practices and procedures of the junior high school and, for that matter, all schools on all levels? Does it have implications for the course of study? Should we not interpret the course of study in terms of the readiness of the pupils in a particular class in a given year? This means more than just providing more work for the brighter pupils and fewer examples for the slower ones. The old plan, you know, used to be to follow the textbook and to give to the faster moving pupils, in addition to the regular assignments, the starred examples also. Youngsters were not long in discovering that it did not pay to be bright. It seems to me that a course of study should contain different kinds of material for the brighter pupils with a different emphasis and not just more of the same kind of work. Somewhere in the course of study there should be provision made for the average and slow-learning pupils, with the emphasis perhaps on social usage. The course of study should also make provision for the teaching of arithmetic as a science to those youngsters who are demonstrating that they have an aptitude for the subject.

Does the degree of readiness have implications for methods of teaching? It is my experience, and perhaps yours, that the youngster who is not ready for the program of a given grade can be helped by building up with him a background of number experiences. This can be done by correlating his experiences during the mathematics period with experiences in other subject matter fields and with his out-of-school experiences. It seems to me that for some time the teacher will steer

clear as much as possible from the teaching of abstractions to this kind of youngster. Research studies have shown over and over again that such children need more explanation and more concrete illustrations of new concepts than do the children who have demonstrated an aptitude for mathematics.

It seems to me that a situation in a class where there is a wide range of maturity and readiness for the program in mathematics requires that each class be divided into groups according to their degrees of readiness. It seems that the best advantage of this procedure is in the possibility of varying the type of material and the rate of progress according to the ability and readiness of each group.

It also seems to me that the wide range in readiness in a given class has implications for the selection and use of materials for the group. I do not believe that there exists a book which will provide the amount of practice needed for some pupils in a class which has a range of four years in achievement. How *can* one book meet the needs of all the youngsters in a class where the range in reading ability is 6 years? That kind of book has probably not been written.

Can any teacher put into practice some of the procedures which are suggested in the discussion above? I believe so. What I am going to present now is one approach to the problem which has been suggested here. One teacher of junior high school mathematics carried a load of four classes with about 35 pupils in a class. After the second week of school a series of diagnostic arithmetic tests was administered to the pupils in these classes. It was interesting to note that in one class, which I remember very well, the diagnostic test in division of fractions resulted in perfect score of 25 for 8 pupils and a low score of 4 for 2 pupils. I give this as an example of a kind of situation with which many 7th grade teachers are faced. On the basis of these diagnostic tests each class was roughtly broken up into three or four groups, which were called *W—X—Y* and *Z*. 7–1a had three groups *W—X* and *Y*; 7–2a and b were divided into four groups *W—X—Y* and *Z*, and 7–2c was divided into group *X*, *Y* and *Z*. Each group began the year's work on the level at which the test showed it was succeeding. Throughout the year as individual youngsters mastered the next step he was moved into a group which was then doing the work for which he had demonstrated he was ready. Obviously some of the youngsters needed more practice material than others and since the textbook which were were supposed to be using did not provide enough practice material for the slower moving groups it was necessary to purchase four or five different kinds of work books, about 12 or 15 of each kind.

Conferences were held from time to time between the mathematics teacher and the teachers in other fields who also worked with these same classes. Plans for correlation between these subjects fields were made and later carried out. For example when learning how to make graphs of social studies, data from the current unit were used instead of the artificial data presented in the arithmetic textbook. The study of the circle in mathematics was timed to the social studies unit entitled, "The Earth, Its Size and Motions." And so on.

Each group had a pupil leader who was responsible for distributing materials, checking in papers, and for helping the teacher keep what could be a rather complex situation, a smooth running machine.

Teachers on the secondary school level have perhaps hesitated to use this kind of class organization because its description suggests demands on the teachers' time and energy. Moreover, at a first glance, the plan appears to be a bit unwieldy. Actually there is very little more in the way of preparation that the teacher has to make once the program is organized and the pupils understand what their responsibilities are. Instead of planning the work for each class, the teacher merely

plans the work for each group. Assuming that a teacher meets five classes a day there would probably be five preparations involved. Should he use the group plan described above he probably would have no more than five preparations and it is possible that he may have only three or four to make depending upon the range in readiness of the pupils assigned to him.

I do not propose this plan as the panacea for all the ills to which teachers of mathematics fall heir. It seems to me, however, to have certain strengths which recom mend itself as one approach to the solutio of this problem of pupil readiness for th next step in mathematics. It does n follow, however, that because the pla seems to work in one school situation tha it will work in all situations. The succe of any plan will depend largely upon a understanding of the needs of the pupil and the degree to which efficient practic can be brought to bear on the learnin situation.

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A General Method for the Construction of a Mechanical Inversor

By M. H. AHRENDT
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IN HIS BOOK, *Linkages: The Different Forms and Uses of Articulated Links*, DeRoos stated the following theorem: "In any quadrilateral whose diagonals may intersect at an angle, a point A of a diagonal can always be taken so related to two of the sides that O, A, and B shall be in a line and so that the product of OA and BA shall be a constant." DeRoos did nothing more in his book, however, than merely state the theorem. He offered no proof of its truth and gave no specifications for the construction of such a linkage. In searching through the literature on linkages, the writer has been unable to find any further mention of this theorem.

A proof of DeRoos's theorem would be of interest in the construction and use of plane linkages. Such a linkage as he suggests would, of course, be an inversor, two of the collinear points always being mutually inverse with respect to the third. A number of such inversors have been devised, but they must all be constructed in keeping with certain limitations. In view of the important uses to which mechanical inversors are put in the construction of linkages, it would be an advantage to have a general method such as DeRoos suggested.

It is possible to demonstrate that the above theorem is true. In fact we can make the theorem even more general. *In any quadrilateral we can, in an unlimited number of ways, add two bars so as to produce three collinear points so related that the product of the distances of two of the points from the third is always a constant.* Thus DeRoos's requirement that the diagonals must intersect is not necessary. The diagonals may be parallel. To find the specifications for constructing such a linkage, it will be convenient to begin with the first known inversor, the Peaucellier cell, and work out some generalizations from it.

The Peaucellier cell, devised in 1864 by A. Peaucellier, an officer in the French army, is simple in construction, consisting of a rhombus, as shown in Figure 1, with two other equal bars longer or shorter than those in the rhombus. Now in the figures we note the following facts:

$$\overline{OX}^2=\overline{OB}^2-\overline{BX}^2$$

$$\overline{XC}^2=\overline{BC}^2-\overline{BX}^2$$

$$\overline{OX}^2-\overline{XC}^2=\overline{OB}^2-\overline{BC}^2$$

$$(OX-XC)(OX+XC)=\overline{OB}^2-\overline{BC}^2.$$

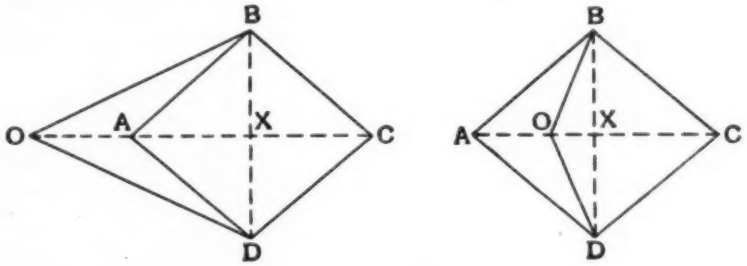

FIG. 1

But

$$XC=AX$$

(1) $\therefore OA\cdot OC=\overline{OB}^2-\overline{BC}^2=\pm$ a constant.

In the second figure we must think of the distance OA as being negative. Thus the numerical value of the constant, when the two equal bars are shorter than those of

the rhombus, is negative, and we have the so-called negative form of the cell.

Before proceeding further, it might be worth while to illustrate one use of mechanical inversors. In Figure 2 let us fix

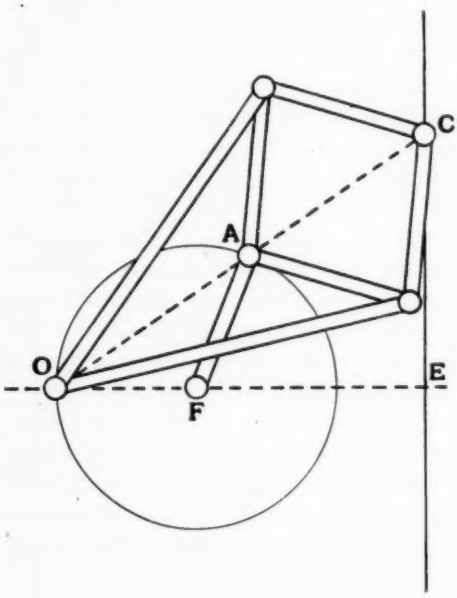

FIG. 2

the point O and by means of a radius bar FA force A to follow an arc of a circle which passes through O. We know from the geometry of inversion that the inverse of any circle passing through the center of inversion is a straight line. Thus C must trace the inverse of this circle described by A, or the straight line CE. It is evident that we can lay off all the distances needed in the construction of this linkage with compasses only. We need not use a straight line in any way in its construction. Thus we have solved the problem of drawing an original straight line. The invention of the Peaucellier cell provided the first satisfactory means for solving this problem.

Now let us study the Peaucellier cell to see what changes might be made in its construction. Kempe pointed out that the cell could be thought of as being composed of two primary elements called a kite and a spearhead, shown in Figure 3. If we superimpose the kite upon the spearhead by placing A upon C and amalgamating the long bars, we have the positive form of the Peaucellier cell. If we make B and D coincide and amalgamate the short bars, we have the negative form of the cell. In fact, it is apparent that any method we can use to join the two elements so that the indicated points are collinear and the two elements are equally deformed (keeping the corresponding angles equal) will give us a linkage with the same characteristics as those of the standard cell.

Kempe further showed that we can change the size of either the kite or spearhead. Suppose we halve the size of the spearhead but keep its dimensions proportional to those of the kite. Evidently, when we connect the kite and the spearhead as mentioned above, the product of AB and CD will still be a constant, but the value of the constant will be just half of what it was before. This second modification gives us quite a variety of new cells, such as those illustrated in Figure 4.

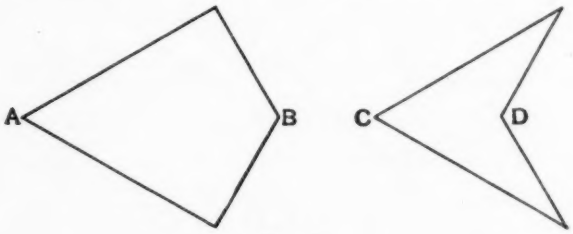

FIG. 3

Still a third modification of the original Peaucellier cell can be made. We have found in equation (1) the power (that is, the value of the constant) of the cell. We see that the power of the cell, $\overline{OB}^2 - \overline{BC}^2$,

depends on the relationship between the bars and not solely on the individual lengths of the bars. That is, two cells may

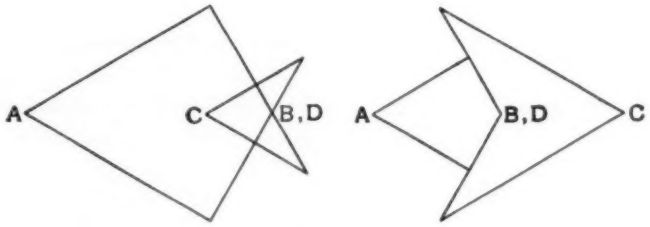

FIG. 4

have the same power and therefore perform identical operations without having bars of equal lengths. If we superimpose one of these cells upon the other, the three collinear points of one must coincide in all positions of the cells with the three points of the other. Apparently then, as in Figure 5, we could attach the upper

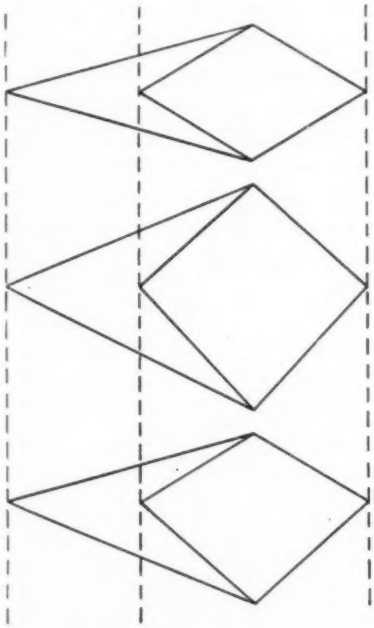

FIG. 5

half of one of these cells to the lower half of the other, and the new cell will have the same characteristics as the unmodified cell. Evidently this same elongation of one side of the cell can be applied also to any of the other modifications already mentioned. The only requirements will be that the kite and spearhead must be proportional and the diagonals of each must intersect at right angles.

Now using a spearhead somewhat smaller than the kite and elongating the lower half of each, we get the linkage shown in Figure 6. For the sake of convenience let us represent the bars and segments of bars by the letters a, b, c, d, e, f, g, and h. If the standard cell had been used with the lower half elongated, we should have the linkage $OBCDZ$. Evidently $BZ = BC = b$, and $DZ = DC = c$. We have reduced the size of the spearhead,

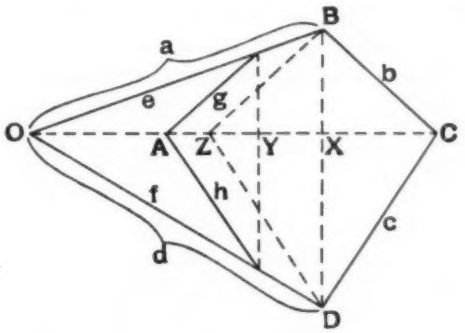

FIG. 6

giving it the sides e and f. Now, through similarity of triangles,

$$\frac{e}{a} = \frac{g}{BZ} = \frac{g}{b}$$

(2) $$\therefore g = \frac{be}{a}.$$

Similarly

(3) $$h = \frac{cf}{d}.$$

We know from equation (1) that

$$OZ \cdot OC = a^2 - b^2.$$

But

$$\frac{e}{OA} = \frac{a}{OZ}$$

$$\therefore OZ = \frac{a(OA)}{e}.$$

Thus

$$\frac{a(OA)(OC)}{e} = a^2 - b^2$$

$$(4) \qquad OA \cdot OC = \frac{e}{a}(a^2 - b^2).$$

Similarly

$$(5) \qquad OA \cdot OC = \frac{f}{d}(d^2 - c^2).$$

Thus we have found the value of the constant or power of this new cell in terms of its link dimensions.

We are now ready to attack directly the DeRoos theorem. Let us suppose that we are given any quadrilateral having sides a, b, c, and d, as shown in Figure 7. Our problem is to determine the lengths of the bars g and h and their points of connection on a and d. Let us treat the upper and lower halves of this cell as the upper and lower parts of two separate modified cells

$$\frac{e}{a}(a^2 - b^2) = \frac{f}{d}(d^2 - c^2)$$

$$(6) \qquad \therefore \frac{e}{f} = \frac{a(d^2 - c^2)}{d(a^2 - b^2)}.$$

Thus we have found the ratio of e to f. Obviously when we are constructing a cell by the use of this ratio, either e or f can

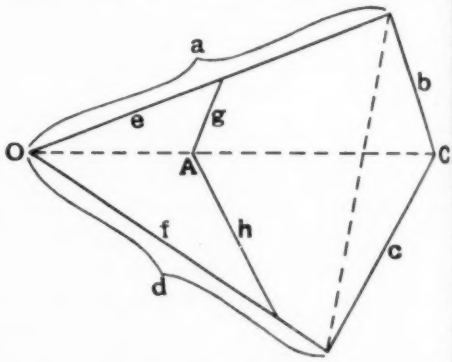

FIG. 7

be assigned arbitrarily. For the sake of convenience and simplicity it will be advantageous whenever possible for us to let $e = a$ or $f = d$. Having determined e and f, we find the lengths of g and h from equations (2) and (3). Thus it is evident from the nature of their derivation that when equations (2), (3), and (6) are applied to

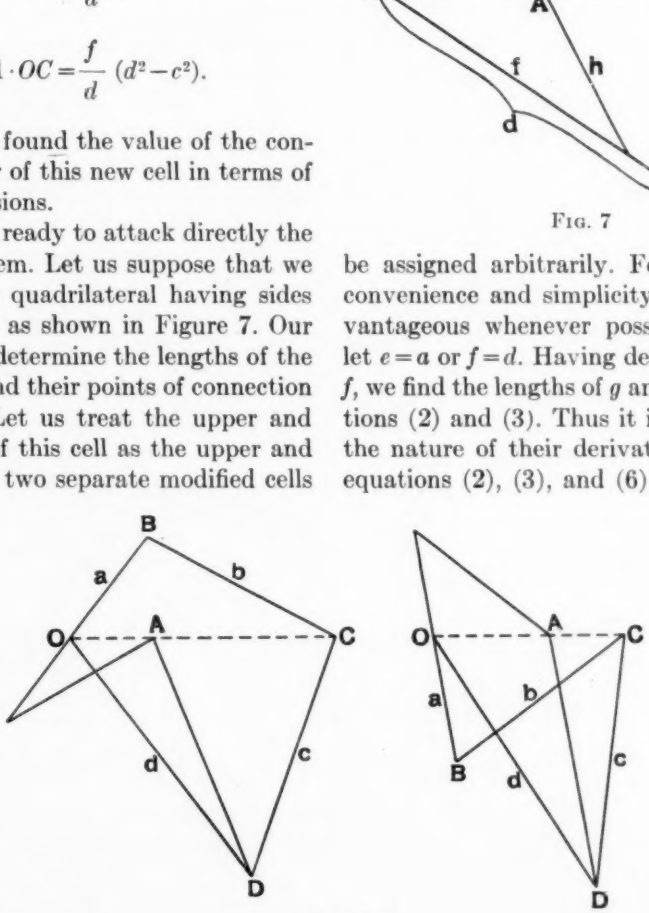

FIG. 8

of the same power. If we put these two halves together, then g and h will necessarily meet at the point A, and A must be collinear with O and C. Also from equations (4) and (5) it will be apparent that: any given quadrilateral, they will enable us to construct from it a mechanical inversor. Our liberty to select either e or f arbitrarily and to begin with any one of the four vertices gives us literally an un-

limited number of ways of handling any given quadrilateral.

Now having arrived at a solution to the problem, it will be interesting to apply it to a number of quadrilaterals. We take a random case as in Figure 8 in which we have sides of 3, 5, 6, and 7 units. We may take these in any order; for instance, $a=3$, $b=5$, $c=6$, and $d=7$. Evidently the quadrilateral may be placed in either an open or crossed position. To show that the method is perfectly general, let us handle both cases. Suppose $f=d=7$. Then the extra bar h will be attached to the quadrilateral at D. By substituting in equations (6), (2), and (3) we find that $e=-39/16$, $g=-65/16$, and $h=6$. The negative values for e and g make it necessary for us to think of the lengths of the bars as directed distances. Evidently laying off e along a from O toward B would correspond to a positive value of e. Negative e must then mean an *extension* of the bar a through O away from B. Thus by comparison with a cell using all positive numbers, as in Figure 7, we can readily interpret any negative values that arise. The cell derived in Figure 8 will perform the same operations as those of the standard cell.

We may next apply our general method to a kite. If a and d are the two longer sides of a kite and if $e=a$, we obtain the positive form of the standard Peaucellier cell. If we let a and d be the two shorter sides and $e=a$, we get the negative form of the cell. If we vary e, we may derive all the possible combinations of an unelongated kite and spearhead.

It will be noted that in one case this general method will produce only a theoretical linkage; that is, when $a=b$ and $c=d$. We see from equation (6) that in this case the ratio of e to f is indeterminate. Thus e and f can be given independently any values. But we find that when g and h are added, they meet at the point O. That is A and O coincide in all positions of the cell, and the power of the cell is zero. Thus, although the cell is theoretically possible, it is mechanically unworkable. Evidently the solution is to rotate the quadrilateral so as to make the point O fall at another vertex. Even this idea, however, fails when the given quadrilateral is a rhombus. The inability to apply this general method to the rhombus is not serious, however, as we have already shown how to derive the only forms of the cell of which a rhombus could be a part, the positive and negative forms of the standard cell.

One quadrilateral is of sufficient interest to deserve special comment. Suppose we begin with a quadrilateral in which the diagonals do not intersect in any position of the quadrilateral. Such a quadrilateral would have to be a contra-parallelogram or a crossed parallelogram, and the diagonals (that is, the bases of the trapezoid *OBDC* of Figure 9) would be parallel. If we apply equations (2), (3), and (6) to a contra-parallelogram, making a one of the short sides and letting $e=a$, we obtain the first linkage in Figure 9, the product of OA and OC being a constant. Now we note in the figure that $ABDO$ is a parallelogram. If we consider directed distances,

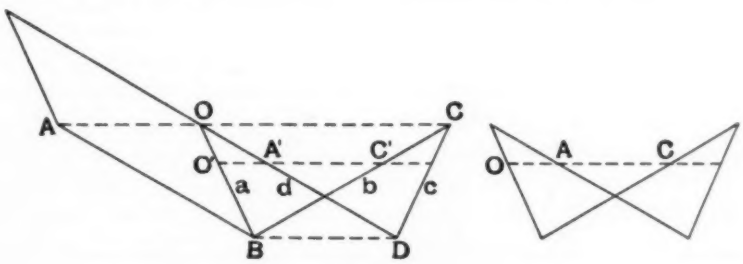

FIG. 9

$$OA = -BD.$$

But

$$(OA)(OC) = -k, \text{ a constant.}$$

$$\therefore (BD)(OC) = k.$$

Now let O', A', and C' be points on a, d, and b lying on a line parallel to OC and BD. Then in the triangles thus formed,

$$\frac{O'A'}{OO'} = \frac{BD}{OB}$$

$$\frac{O'C'}{O'B} = \frac{OC}{OB}$$

$$\therefore \frac{(O'A')(O'C')}{(OO')(O'B)} = \frac{(BD)(OC)}{(OB)(OB)}$$

$$(O'A')(O'C') = (BD)(OC)\ \frac{(OO')(O'B)}{(OB)(OB)}$$

$$= k\ \frac{(OO')(O'B)}{(OB)(OB)}$$

$$= \text{a constant.}$$

Therefore, if we use instead of O, A, and C the points O', A', and C', we may eliminate the two extra bars and the extension of d, using just the original quadrilateral. Thus we have derived the mechanical inversor discovered by Hart. In Hart's inversor, O, A, and C may fall on any line parallel to the diagonals of the contraparallelogram.

Having now determined the specifications for this generalized cell, the question arises as to how we shall actually construct it. Obviously it is necessary that the construction of the cell must not require the use of a straight line, for the drawing of the straight line is one of the important problems which this linkage enables us to solve. If we must have the straight line before we construct the linkage, then the linkage does not solve the problem. Using some of the constructions for the compasses by the Italian geometer, Mascheroni, it can be demonstrated that it is possible to construct this cell with the compasses alone. Given only the end points of any four line segments we can locate with the compasses alone all the other points needed to construct the cell. It is not necessary that we be able to draw the straight line connecting any of the points. In cutting the bars for the cell, we shall, for the sake of appearance perhaps, want to make the bars as straight as possible. This procedure is not in any sense necessary, however, to the operation of the cell. Any inextensible plane pieces will do for the bars. It is only necessary that the material be sufficiently rigid that the fixed distances between pivots remain constant.

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Ungraded Classes for Superior Pupils

By BURR D. COE
Monroe High School, Rochester, New York

ELEMENTARY algebra, plane geometry, intermediate algebra, plane and spherical trigonometry, solid geometry, and advanced algebra are all being studied in the same room at the same time. Sounds something like a one-room country school, doesn't it? This is being done by a group of mentally superior pupils in two ungraded classes (taught by the writer) at Monroe High School.

We have had special classes for superior pupils at Monroe High School for many years. Beginning in January 1939, the best of the superior pupils were placed in special honor classes, called Honor Work Classes, in English, mathematics, science, social studies, and French. We have two ungraded sections of about sixty such pupils, from the ninth through the twelfth grade, meeting in specially equipped rooms. The pupils are selected on the basis of intelligence tests and must have a past record of superior school work. The purpose of the program is to give those pupils who have unusual mental powers a chance to develop habits of independent study, responsibility, and to study without the inevitable restrictions placed upon such pupils in the average class. They are free to proceed at their own rate, to use individual initiative, and to develop special talents; in short, they are given every encouragement to utilize their unusual gifts to the fullest.

It was obvious from the start that ordinary classroom methods would not be applicable. We found that methods which succeeded in the mathematics classes were unsuccessful in other subjects. We have been using a modified form of the seminar method in the mathematics classes. While these methods have been quite successful we shall continue to try to devise means for encouraging the gifted pupil.

Since these pupils all go on to college, or prepare for college, we follow the general syllabus laid down by the New York State Board of Regents. The Regents syllabus is merely the beginning for this group, however, and they all study considerably beyond these minimum requirements. They all take the Regents examinations but these examinations are, on the whole, too easy for them, and do not offer a sufficient challenge. It is not uncommon for the group of intermediate algebra students, for example, to average better than 98% on a Regents examination.

All of the pupils study elementary algebra, plane geometry, and intermediate algebra. Trigonometry, solid geometry, and advanced algebra are elective. Most of them elect trigonometry but only those especially interested in mathematics or who are going on to technical schools elect solid geometry and advanced algebra.

To supplement the Regents syllabus we use all the "optional" material in the usual texts. We use workbooks in elementary algebra and plane geometry and have an extensive classroom library of enrichment materials. Our library consists of nearly one hundred volumes covering all phases of mathematical knowledge and applications. We make extensive use of many types of visual aids, slide rules, surveying and navigation instruments, and periodicals.

If you should walk into the classroom on a typical day this is what you would find: A small group of students would be sitting around a table at the front of the room with the teacher discussing some phase of the work. The rest of the pupils would be working at their desks, frequently two or three together but more often independently. In general, no two pupils would be working on exactly the same thing since they progress at varying rates. The group at the table likewise

would not all be equally well-informed concerning the topic under discussion. For example, it might be the trigonometry group discussing polar co-ordinates. Some of the group would have previously studied polar co-ordinates, some might be in the process, and still others might never have heard of them, except in a casual way. The discussion is one wherein the pupils ask the questions, the teacher advises and helps the group to pick out salient facts, points out important applications, and suggests supplementary reading. There is no recitation in the sense of the teacher quizzing the students. It is taken for granted that the pupil knows the subject or soon will. The discussion is informal and frequently other pupils who are not studying that particular subject will join the discussion and they are always welcomed. In general, each day the teacher will be working with a different group, but with the younger pupils more often than with the older ones. After four years in this environment, the twelfth graders are largely self-sufficient and only infrequent conferences are necessary.

There is no homework or report cards in the usual sense. Each pupil plans, checks, and evaluates his own work; with the exception of topic tests, prepared and corrected by the teacher, but taken as an individual whenever he is ready for them. Master week-by-week schedules are posted on the bulletin board to aid the pupil in his planning. Considerable effort is made, particularly with the younger pupils, in aiding them to budget their time, select pertinent study material, and to organize their work. It was found best to insist that each pupil keep up to this minimum schedule, but most of them are considerably beyond it. Some will accelerate considerably, but on the whole they do not accelerate as much as one might expect because they find that it takes longer to "dig it out for themselves."

At regular intervals the teacher sits down with each pupil and talks over his progress with him. His strong and weak points are discussed and together plans are worked out for improvement. At that time the teacher counsels with him concerning all phases of the work, not just subject matter accomplishment, but study habits, attitude, and cooperation. When a parent wishes reassurance, beyond what the pupil may tell him verbally, it is accomplished by a personal visit to the school rather than by a formal written report. At the end of the school year a report is sent home summarizing the year's accomplishments. At first the pupils missed the "crutch" of an "A" report card but by now neither pupils nor parents seem very much concerned with marks as such.

Each pupil decides for himself how much and what mathematics (if any) he will study on a particular day. After he has prepared his work he checks it himself with answer books always available for his use. If he needs individual help he goes to the teacher or to one of the older pupils. We have stressed the responsibility of the older pupils for their younger companions. This has a two-fold advantage; it keeps the older ones on their toes mathematically and develops a social sense of responsibility for others.

It is not our purpose to get these pupils into college younger or to do college mathematics in the high school. We do not urge acceleration but stress enrichment. The enrichment is not "dragged in by the scruff of the neck," but is made purposeful. All additional work is voluntary and directed along lines of the pupil's special interests. For example, a girl interested in history wrote an excellent paper on the life of Descartes, a boy interested in drafting prepared a remarkable collection of solid geometry figures, a ninth grade boy developed an interest in the slide rule and mastered several different types, a boy interested in biology spent several hours trying to develop mathematical formulas for the laws of

heredity. This is the type of enrichment which is self-starting, useful, and stimulating.

Since the war, stress has been placed on the mathematical needs of the nation. Spherical trigonometry with emphasis on naval and military applications has been added. While we have always stressed the development of a thorough mathematical foundation as our first concern, we bring in "war" applications wherever they can be reasonably applied.

From the point of view of subject matter mastery, the Honor Work Classes have been very successful. The pupils attain very high marks on all examinations, stand out on all types of standardized achievement tests, and begin college mathematics with a considerable advantage over the average freshman. We do not consider this the principal function of such classes for mentally superior students, however. The development of more effective study habits, more self-reliance, a greater sense of responsibility for accomplishment and conduct, a better all-round personality development, to be of greater service to the school community, these are some of the more basic objectives of our program. We believe we are attaining these goals; our own observations and the achievements of those who have gone on to college attest to that fact.

THE ALGEBRA OF OMAR KHAYYAM

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The author of the *Rubaiyat* was also an astronomer and mathematician. This work presents for the first time in English a translation of his algebra. In the introduction, Mr. Kasir traces the influence of earlier Greek and Arab achievements in mathematics upon the algebra of Omar Khayyam and in turn the influence of his work upon mathematics in Persia in the Middle Ages. The translation is divided into chapters, and each section is followed by bibliographical and explanatory notes.

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◆ THE ART OF TEACHING ◆

Mathematics and the Armed Service Educational Program

By HALE PICKETT
State Teachers College, West Chester, Pennsylvania

ABOUT THE time of the financial depression the teaching of mathematics also seemed to be at a low tide. Mathematics was being abandoned by State Departments of Education, college requirements, and local high school programs.

Teachers of mathematics were constantly placed on the defense. The subject became unpopular in many respects.

Suddenly, the War Department made its demand for men; trained in all types of mathematics. The Naval and Army Officers were outspoken relative to the mathematical abilities of the servicemen. Since they were recent products of our school systems, we were criticized for their low level of abilities. This deficiency was only a logical result of our previous educational era. In other words, it seemed that it has taken a war to return mathematics to our educational programs.

The teaching of mathematics in our Armed Forces is characterized by the following:

I. It is an accelerated program.
II. It is a practical program.
III. It has a life or death motivation.

The accelerated program implies that only the essentials be taught in a minimum amount of time. Subjects are abbreviated and are taught in accordance with war time demands. One criterion for a unit is: "Does this unit have any practical value for the Armed Forces?" This means that formulas are presented on faith. Exact mathematical procedures are replaced by computing devices, such as the circular slide rule, E6B computer, the reading of graphs, completion of special forms as H.O. 211, and scale drawings. Special training of special students, for small areas of special procedures have reached complimentary results.

How may we as teachers profit by the experimental procedures of the mathematical programs of the Armed Forces?

The solution of the *triangle of velocities* in navigation certainly gives us a practical use for the *Law of Sines* and the *Law of Cosines*. In the future Air Age these problems will be common place and therefore their solutions should be stressed in trigonometry.

Celestial Navigation implies the global world and demands that concepts of Sperical Trigonometry be included in our course. Rates, time, distance, scales and directions seem to need additional emphasis. Distances are expressed in air hours.

Although the accelerated, practical mathematical programs have been producing efficient soldiers, I doubt whether such a program is correct as a preparation for life in this complex society. The training is superficial, procedures are learned for special duties without knowing the underlying principles. They are narrow in scope, and become automatic. For a broad meaningful mathematical training time is needed for assimilation as opposed to an accelerated program. The practical program of our Armed Forces during a war would not be a practical program for a civilian during peace.

The teaching of mils and trajectors although practical now need not be included in a post war curriculum. As a teacher I am envious of the motivation of these service programs which to me is a blend of

patriotism and survival of the fittest. I wish it were possible for teachers of civilians to so select practical units and so present them that the students would be eager to master them; but I should not wish for the extreme "life and death" motivation which permeates these accelerated practical educational programs in our Armed Forces.

Valentine to Mathematics*

Called the Queen of Sciences by all,
Mathematics rules in every hall.
Geometrical lines so full of grace,
Resembling beautiful hand woven lace.
Algebra, a language international,
With logic in every equation real or irrational.
O Mathematics, O Queen of every line,
Will you be my valentine?

* Presented as a Valentine to the Mathematics Club of Maplewood-Richmond Heights High School, Maplewood, Mo. by the author Kenneth Roettger.

Valentine II*

As Algebra, I met you in my freshman year,
My brains your problems then did sear.
They proved a Waterloo to me,
With their familiar x, y, and z.

Next, Geometry slipped me a mickey,
Trying to tell a rhombus from another doo hickey.
A triangle I tried to trisect,
But it required too much of my intellect.

In solid more problems were heaped on my shoulder,
And my head began to smolder.
To see problems, I stretched my imagination,
And my brains I tried to ration.

Advanced Algebra added to my education,
With its binomial theorem and quadratic equation.
For I gave my all and best,
On each and every test.

Then came algebra, fourth year,
And filled me with formulae from ear to ear.
And now trigonometry comes to me,
And triangular minded I will be.

So through the year you will find,
To you, Mathematics, I have been kind.
And so Mathematics mine,
Will you be my Valentine?

* By Kenneth Roettger.

EDITORIALS

PRESIDENT SMITH has asked us to give the following message to the members of The National Council of Teachers of Mathematics:

A Message from the President

It hardly seems two years since I was writing a message at the beginning of my presidency and wondering what was going to happen to the Council in those two years. In another sense it seems much longer than two years, because so much has been crowded into so little time It has been different from any other two years of the Council. For one thing, we have had no national meetings, but we have had several regional meetings. We had several such meetings last year and we have plans for meetings this year in New York, Cleveland, Detroit and Washington. Many of the meetings on the West Coast have been held in close cooperation with the state representatives of the Council.

The most outstanding work we have done has been the two national reports, 'Pre-Induction Courses in Mathematics" and "Essential Mathematics for Minimum Army Needs." I am convinced that these reports satisfied a definite need and helped to focus the efforts of the many committees working on similar problems. We are indebted to the Army, the Navy, the Office of Education and other departments in Washington for their helpful cooperation.

The directors and past presidents of the Council will meet in New York on February 25, 26 and 27 to initiate plans for leadership in the teaching of mathematics in the postwar period. The solution of the problems involved will take the combined intelligence of all of us. If you have any suggestions, do not hesitate to write to me. Tell us about the changes you have made during this war time, what changes you think ought to be permanent, what can be done so that we shall never again find such poor preparation in mathematics as we did at the beginning of the war. (Address me at 16 Highland St. Longmeadow, Mass.)

If you live near New York, I hope that you will attend the regional meeting at Teachers College, 525 West 120 Street, New York, on Saturday, February 26.

ROLLAND R. SMITH

The above message is timely and, in view of the great importance of mathematics in any plans for the postwar world, it is to be hoped that members of the Council will be generous with their suggestions to President Smith. In a recent number of THE MATHEMATICS TEACHER we asked for suggestions as to the scope and nature of the 1944 (the 19th) yearbook. We have had exactly three letters so far from over 5000 members. These are busy times for all of us, but we must not forget to do all that we can to keep mathematics in its rightful place. Otherwise we may expect that after the war is over the enemies of mathematics will be out in force against the subject with the familiar slogans and arguments. This is no time for apathy.

W. D. R.

New Officers for the Next Two Years

BALLOTS have already been sent out by Secretary Schreiber to the members of the Council asking them to mark them and return them to him before February 18, 1944. In years past some members either have not been interested enough to vote for their favorite candidates or they have simply forgotten to do so. It is extremely important for the sake of progress that each member vote each time he has an opportunity. Let us all surprise the Secretary by giving him enough returns to give him some practice in counting.

W. D. R.

The New York Meeting

AS STATED above one of the regional meetings of the National Council of Teachers of Mathematics this year will be held at Teachers College, Columbia University, in New York City, on Saturday, February 26, 1944. Inasmuch as President Smith has announced a meeting of the Board of Directors of the Council for February 25, 26 and 27, this meeting is practically equivalent to an annual meeting as was customary until the advent of the war.

The program of the New York meeting of the Council appears on page 88 of this issue. Let us have a record attendance!

W. D. R.

◆ IN OTHER PERIODICALS ◆

By NATHAN LAZAR

Midwood High School, Brooklyn, New York

School Science and Mathematics

December 1943, vol. 43, no. 9.

1. Schreiber, Edwin W., "Mathematical Snapshots," pp. 795–799.
2. Harper, Jos. P., "An Approximate Euclidian Trisection," pp. 812–816.
3. Nyberg, Joseph A., "Notes from a Mathematics Classroom," (continued), pp. 843–847.
4. Beiler, A. H., "Prolate and Curtate," pp. 848–851.
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1. Andrews, C. L., "Graph of the Lens Equation in Three Variables," *American Journal of Physics*, 11: 293–296, October, 1943.
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6. DeMay, A. J., "Multiplication Rationalized for Grade Three," *Instructor*, 52: 27–28, September, 1943.
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9. Gillam, E. E., "Mechanical Aids in Teaching Junior Mathematics," *School (Secondary Edition)*, 32: 145–149, October, 1943.
10. Grossnickle, F. E., "Kinds of Errors in Division of Decimals and Their Constancy," *Journal of Educational Research*, 37: 110–117, October, 1943.
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15. Hartung, M. L., "A Significant Analysis of Mathematical Needs," *School Review*, 51: 516–518, November, 1943.
16. Hay, S. S., "Preflight Aviation Entrance Examination," *Industrial Arts and Vocational Education*, 32: 398+, November, 1943.
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19. Manheimer, W. P., "Enrichment of the Advanced Algebra Course," *High Points*, 25: 39–47, October, 1943.
20. "Mathematics and Science in Wartime," *Maryland, Department of Education*, Vol. 24, no. 1.
21. Menger, H., "Arithmetic for Individual Needs; Grade III," *School (Elementary Edition)*, 32: 127–128, October, 1943.
22. Professional Background Material for War Mathematics," *Florida Program for Improvement of Schools, Bulletin no. 41.*
23. Richardson, R. G. D., "Mathematics and the War," *The American Scholar*, 12: 503–505, October, 1943.
24. Rosenberg, R. R., "Drills in Preinduction Mathematics," *Business Education World*, 24: 17–18, 105, September–October, 1943.
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Program of the Regional Meeting of the National Council of Teachers of Mathematics

Teachers College, Columbia University
525 West 120th Street
New York 27, N. Y.
February 26, 1944

General Topic

MATHEMATICS IN WAR AND PEACE

Morning Session
10:15–12:15

PANEL I

Theme: Mathematics in the Senior High School.
Place: Room 102 Dodge Hall.
Presiding: Julius H. Hlavaty, Bronx High School of Science, New York City.
Speakers: Max F. Weiss, Midwood High School, Brooklyn, N. Y.; W. D. Reeve, Teachers College, Columbia University, New York City.

PANEL II

Theme: The Role of Mathematics in the Training Program for Adults and Veterans.
Place: Room 211 Teachers College.
Presiding: Solomon C. Greenfield, Administrator, War Industries Training Program, Board of Education, New York City.
Speakers: To be announced.

PANEL III

Theme: Mathematics in Science and Industry.
Place: Room 301 Teachers College.
Presiding: James Naidich, Manhattan High School for Aviation Trades, New York City.
Speakers: To be announced.

PANEL IV

Theme: Social and Economic Aspects of Statistics in the Post-War Period.
Place: Room 327 Macy.
Presiding: W. S. Schlauch, New Yo University.
Speakers: To be announced.

Luncheon

Time: 1 P.M.
Price: $1.00.
Place: Men's Faculty Club of Columb University, Morningside Drive a 117th St.
Presiding: Rolland R. Smith, President the National Council of Teachers Mathematics, Springfield, Mass.
Guest Speaker: Warren W. Knox, Dir tor of Secondary Education, Ne York State Education Departme Albany, N. Y.
Topic: The Place of Mathematics in Se ondary School Programs.

Reservations for this luncheon must made not later than Monday, Febru 21, 1944. If you are not a member of a of the three cooperating organizatio listed below, send your reservations (nam and address only, as payment can made at the door) to Dr. Nathan Laz Midwood High School, Bedford Aven and Glenwood Road, Brooklyn, N. Y.

Cooperating Organizations

1. Association of Chairmen of Depar ments of Mathematics of New Yo City.
2. Section 19 (Mathematics) of the Ne York Society for the Experimen Study of Education.

3. Association of Teachers of Mathematics of New York City.

Program Committee

Benjamin Braverman, Seward Park High School, New York City.
Eugenie C. Hausle, James Monroe High School, New York City.
Max Peters, New Utrecht High School, Brooklyn, N. Y.
William L. Schaaf, Brooklyn College, Brooklyn, N. Y.
Nathan Lazar, Chairman, Midwood High School, Brooklyn, N. Y.

14th Yearbook

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by IVAN STEWART TURNER, Ph.D.

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